

Exam 1 – Kinematics and Force

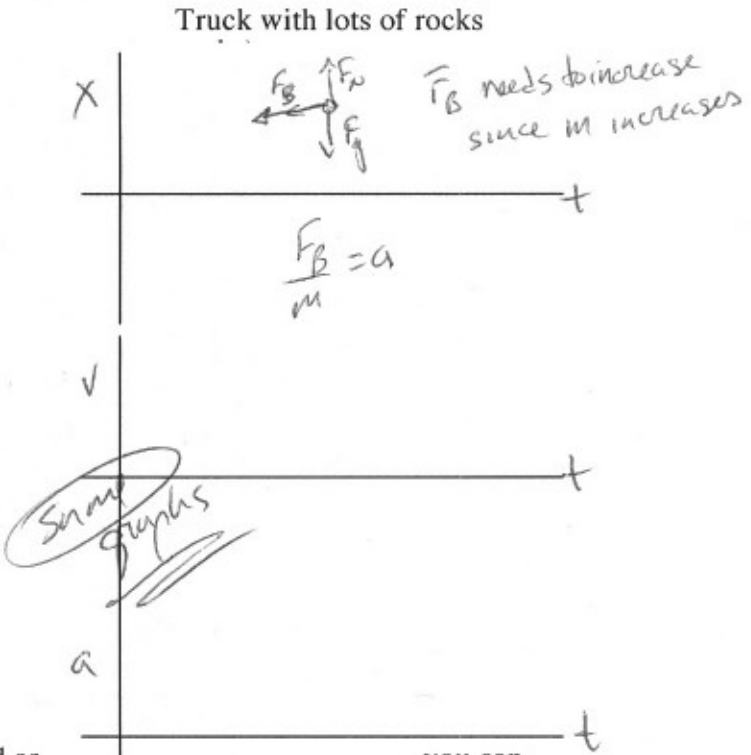
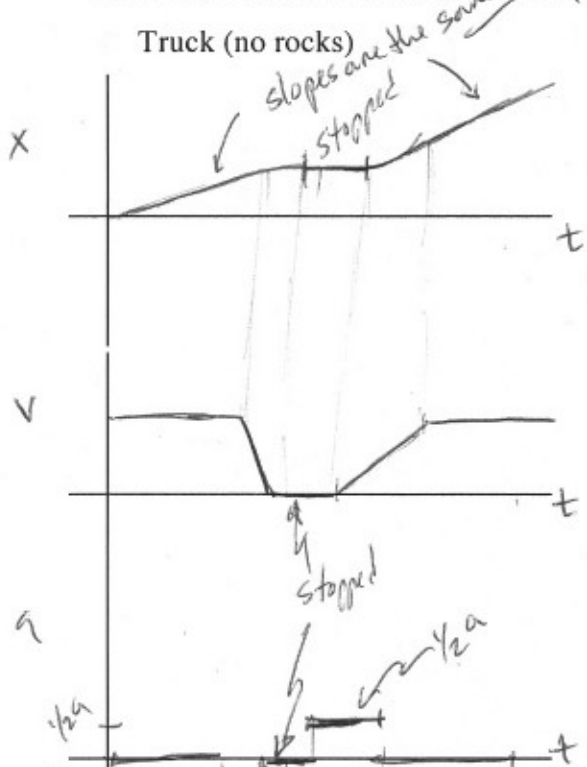
September 24, 2008

This is a closed book examination. There is extra scratch paper available. Your explanation is worth 3/4 of the points. Explain your answers!

A general reminder about problem solving:

1. Draw a picture then create a simplified free body diagram with all forces
2. Write down what you know including coordinate frame
3. Write down what you don't know and/or want to know
4. List mathematical relationships
5. Simplify and solve
6. Check your answer – Is it reasonable? Are units correct?
 - Show all work!

1. [24 PTS] You are driving your truck down the street at a constant velocity and approach a stop sign. You slow down, stop and then resume driving down the street at the same constant velocity. The magnitude of your acceleration when you are stopping is twice the starting acceleration. Neglecting friction, draw $x(t)$, $v_x(t)$ and $a_x(t)$ for an empty truck and for a truck carrying a full load of rocks. Indicate when the truck is stopped and when the truck is moving at a constant velocity on each graph. Indicate what changes on the graphs when the mass of the truck increases (assume that the initial velocity is the same)

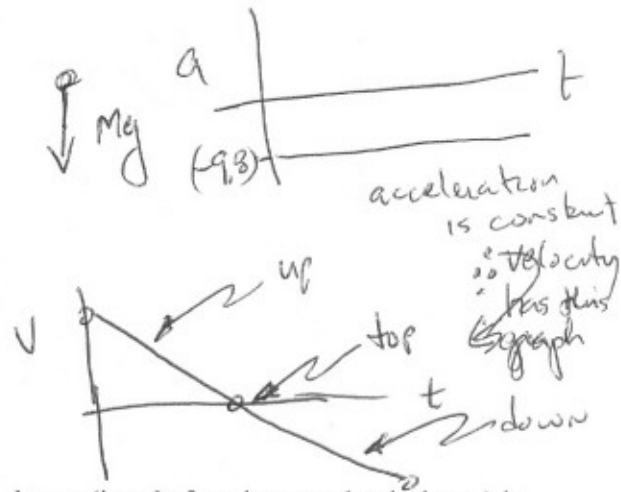


You throw a heavy small yellow ball as hard as straight up into the air. Assume up is the positive refer to this ball after it has left your hand. Please explanation is worth 3/4 of the points).

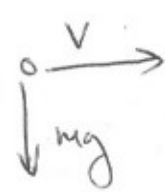
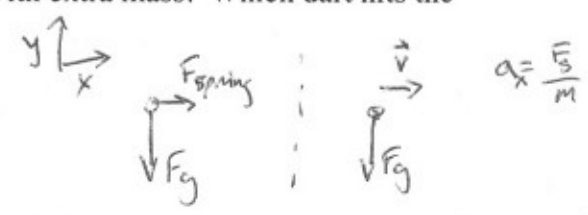
you can direction. The next six questions explain your answers (your

2. [4 PTS] The acceleration of the ball on the way up is

- a) 9.81 m/s^2 in the upward direction.
 b) zero (no acceleration).
 c) 9.81 m/s^2 in the downward direction.
 d) Can not tell. It depends on the initial velocity.
3. [4 PTS] The velocity of the ball on the way up is
 a) positive (in the upward direction).
 b) zero.
 c) negative (in the downward direction).
 d) Can not tell. It depends on the initial position.
4. [4 PTS] The acceleration of the ball at the very top of its throw (just before it starts back down) is
 a) 9.81 m/s^2 in the upward direction.
 b) zero (no acceleration).
 c) 9.81 m/s^2 in the downward direction.
 d) Can not tell. It depends on how high it was thrown.
5. [4 PTS] The velocity of the ball at the very top of its throw (just before it starts back down) is
 a) positive (in the upward direction).
 b) zero.
 c) negative (in the downward direction).
 d) Can not tell. It depends on the initial position.
6. [4 PTS] The acceleration of the ball on the way down is
 a) 9.81 m/s^2 in the upward direction.
 b) zero (no acceleration).
 c) 9.81 m/s^2 in the downward direction.
 d) Can not tell. It depends on where you catch it.
7. [4 PTS] The velocity of the ball on the way down is
 a) positive (in the upward direction).
 b) zero.
 c) negative (in the downward direction).
 d) Can not tell. It depends on the initial position.



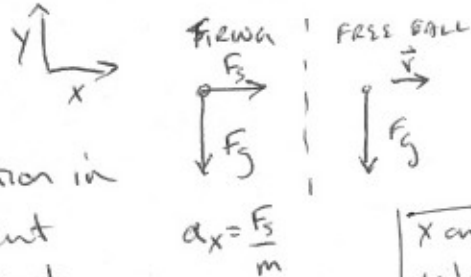
8. [4 PTS] Two identical spring-loaded dart guns are fired horizontally at the same time. One fires a regular dart while the other a dart with extra mass. Which dart hits the ground first?
 a) The regular dart.
 b) The dart with extra mass.
 c) Both darts hit the ground at the same time.



Both darts have same ^{vertical (y)} acceleration ~~down~~ (g)
 They have different horizontal (x) acceleration but that just means they hit the ground at the same time but at different ~~pos~~ distances.

8. [4 PTS] Two identical spring-loaded dart guns are fired horizontally at the same time. One fires a regular dart while the other a dart with extra mass. Which dart hits the ground first?

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Both darts have same acceleration in the vertical (g). They have different horizontal (x) acceleration but that just means they hit the ground at the same time but at different distances

$$a_x = \frac{F_s}{m}$$

x and y are independent

9. [4 PTS] A deer runs out of the woods and hits the side of your car. The deer is much less massive than your car. It follows that the force the car exerts on the deer

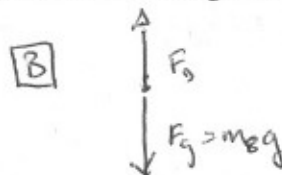
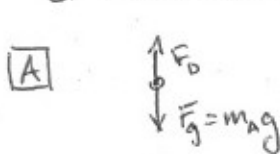
- a) is greater in magnitude than the force the deer exerts on the car.
 b) is the same magnitude that the force the deer exerts on the car.
 c) is smaller in magnitude than the force the deer exerts on the car.
 d) can not be compared to the force the deer exerts on the car since you do not know the angle of impact.

$$\vec{F}_{\text{deer}} = -\vec{F}_{\text{car}}$$

Same but opposite forces - Not because the objects have different masses they experience different accelerations

10. [4 PTS] Two objects are dropped from a hot air balloon. The objects are identical except $m_B > m_A$. The first object ("A") reaches terminal velocity much faster than the second object ("B"). It follows that force due to air drag on object A is

- a) greater than the force due to air drag on object B.
 b) the same as the force due to air drag on object B
 c) smaller than the force due to air drag on object B



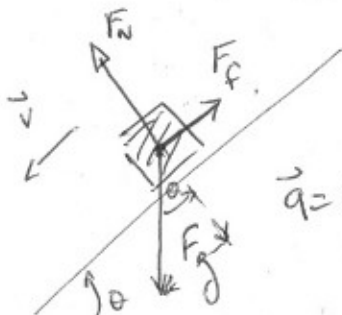
NOTE: $F_D \propto v^2$ so $v_B > v_A$

$\Delta v = 0, a = 0$ so $F_{\text{NET}} = 0$

$F_D = F_g = mg$

$F_{DA} < F_{DB}$ since $m_A g < m_B g$

11. [6 PTS] Draw the force diagram for a large box sliding down a steep incline at constant velocity. Pay attention to the magnitude of your force vectors. Add acceleration and velocity vectors if they are appropriate. Include friction.



NOTE: $|F_g \cos \theta| = |F_N|$ and $|F_g \sin \theta| = |F_f|$

$\vec{a} = 0$. Which means $\sum \vec{F} = 0$ box is not accelerating (\vec{v} is constant, $\Delta \vec{v} = 0$)

Please do the next two problems on additional paper or on problem solving sheets.

12. [12 PTS] While visiting New York City you climb to the 102nd floor of the Empire State Building. Standing on the observation deck you wonder how fast an object would be traveling when it hit the ground if it were dropped over the railing. You notice that the sign says that the observation deck is 1224 feet (373 m) above the street.
- Ignoring air drag how fast would a penny (mass = 6 grams) be traveling when it hit the ground? Assume you drop the penny.
 - How long would it take to hit the ground?
Instead of dropping the penny you now throw it with a horizontal velocity of 2 m/s.
 - How long does it take the penny to hit the ground?
 - How far away from the building does the penny land?
13. [12 PTS] If you can “throw” a 14 lb (6.4 kg) shot put 32 feet (9.75 m), how far can you “throw” a 12 lb (5.4 kg) shot put? Assume your throwing technique is identical for each shot put which means your applied force is the same and the distance you apply the force (i.e. the length of your arm) is the same.

Possibly useful mathematical relationships:

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2 \cos^2(\theta) - 1 = 1 - 2 \sin^2(\theta)$$

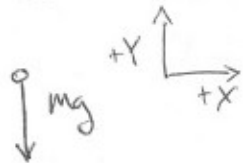
Derivative of a polynomial $\frac{d}{du} Cu^n = nCu^{n-1}$

Anti-derivative (integral) of a polynomial $\int Cu^n du = \frac{1}{n+1} Cu^{n+1} + const.$

The Chain Rule $\frac{d}{dz} f(u) = \frac{d}{dz} u \frac{d}{du} f(u)$

Assume the acceleration due to gravity is: $g = 9.8 \text{ m/s}^2$

$$\boxed{12} \quad \sum F = ma$$



$$\int a dt = v(t) = at + v_0$$

$$\int v(t) dt = x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

Ignore air drag $ma_y = mg$ $a_y = -g$ for penny (or any object!)

$$v_0 = 0 \text{ m/s}$$

$$x_0 = 373 \text{ m}$$

$$a_y = -9.8 \text{ m/s}^2$$

$$0 \text{ m} = \frac{1}{2}(-g)t^2 + 373 \text{ m} \quad \therefore t = \left(\frac{(373 \text{ m})^2}{9.8 \text{ m/s}^2} \right)^{1/2}$$

$$t = (76 \text{ sec}^2)^{1/2} = 8.7 \text{ s}$$

$$v(8.7 \text{ s}) = -9.8 \frac{\text{m}}{\text{s}^2} (8.7 \text{ s})$$

$$= -86 \text{ m/s}$$

$$(a) -86 \text{ m/s} = v_y$$

$$(b) 8.7 \text{ s} = \Delta t$$

Throwing penny with horizontal velocity does not change any of the vertical starting conditions

$$(c) 8.7 \text{ s} = \Delta t$$

$$(d) 17.4 \text{ m} = \Delta x$$

$$x(t) = \frac{1}{2}a_x t^2 + v_{0x}t + x_0$$

$$x(8.7 \text{ s}) = \frac{1}{2}a_x (8.7 \text{ s})^2 = 17.4 \text{ m}$$

$$x_0 = 0 \text{ m}$$

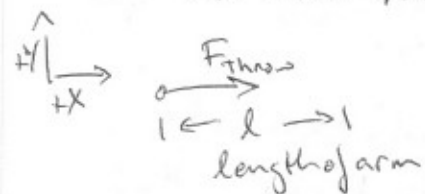
$$a_x = 0 \text{ m/s}^2$$

$$v_{0x} = 2 \text{ m/s}$$

$\boxed{13}$ Want distance of throw (Δx).

Assume horizontal throw \Rightarrow time in air is the same so $\Delta x = vt$

\Rightarrow Need velocity at end of throw



$$x(t) = x_0 + v_0t + \frac{1}{2}at^2$$

$$v(t) = v_0 + at$$

$$\therefore 2la = v^2$$

$$v_0 = 0$$

$$t = \frac{v - v_0}{a} = \frac{v}{a} \quad x - x_0 = l = \frac{1}{2}a\left(\frac{v}{a}\right)^2$$

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F} = m\vec{a} \quad v_x = \left(2l \frac{F}{m}\right)^{1/2}$$

$$\text{mass ①} \quad v_1 = \left(\frac{2lF}{m_1}\right)^{1/2}$$

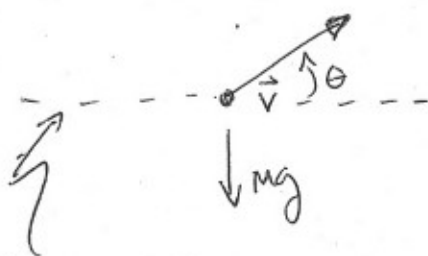
$$\text{mass ②} \quad v_2 = \left(\frac{2lF}{m_2}\right)^{1/2}$$

$$\frac{v_1}{v_2} = \left(\frac{m_2}{m_1}\right)^{1/2} \quad \text{since } \Delta x = vt \quad \frac{\Delta x_1}{\Delta x_2} = \left(\frac{m_2}{m_1}\right)^{1/2}$$

$$\Delta x_2 = \Delta x_1 \left(\frac{m_1}{m_2}\right)^{1/2}$$

$$9.75 \text{ m} \left(\frac{6.4 \text{ kg}}{5.4 \text{ kg}}\right)^{1/2} = 10.6 \text{ m}$$

If you do not assume the throw is horizontal you need to determine how range is related to initial velocity



Assume mass is thrown from ground (not first time but simpler)

$$v_x = v_0 \cos \theta \quad a_x = 0$$

$$v_y = v_0 \sin \theta \quad a_y = g$$

$$R = v_x t$$

$$\text{Height} = v_y t - \frac{1}{2} g t^2$$

$$R = \frac{v_x v_y 2}{g}$$

$$0 = v_y - g \left(\frac{t}{2} \right) \quad \text{@ top } v = 0$$

$$t = 2 \frac{v_y}{g}$$

$$R = \frac{v_0^2}{g} 2 \sin \theta \cos \theta$$

use trig relationship

$$R = \frac{v_0^2}{g} \sin(2\theta)$$

so $R \propto v^2$

$$\frac{R_1}{R_2} = \left(\frac{v_1}{v_2} \right)^2 = \frac{m_2}{m_1}$$

$$R_2 = R_1 \frac{m_1}{m_2}$$

$$= 9.75 \text{ m} \left(\frac{6.4 \text{ kg}}{5.4 \text{ kg}} \right) = 11.6 \text{ m}$$

Note: This does not take into account the height of the mass when it is released